Detection of Sea-Water Motion by Nuclear Precession

E. L. HAHN

University of California
*LaJolla, California**

The long memory of nuclear-spin Larmor precession can be utilized to detect small changes in precession phase angle. The detection of long-range, slow transport of sea water caused by internal waves or other disturbances would seem desirable. Consider how the transport of a volume element of spins in a liquid through a spatial, inhomogeneous, magnetic field affects the phase of Larmor precession. For simplicity, consider a volume element of spins at position x_0 at time $t = 0$, and assume that this volume element moves with a constant velocity $\mathbf{v}$ in an inhomogeneous field $H(x)$. The magnitude and direction of $\mathbf{v}$ is to be measured, and it is shown that $\mathbf{v}$ as small as 10^{-3} cm/sec is detectable. The free precession signal [*Hahn*, 1950] for this element of spins is defined in terms of the vector

$$V(t) = \exp\left[j\gamma \int_0^t H(x_0 + vt)\, dt\right]$$

where, at time t, $vt = x - x_0$; γ is the spin gyromagnetic ratio. After writing $H(x_0 + vt)$ as a Taylor's series and keeping the first two terms for convenience, we obtain

$$V(t) = \exp\left\{j\gamma \int_0^t \left[H(x_0) + \left.\frac{dH}{dx}\right|_{x_0} vt\right] dt\right\}$$

$$= \exp j\gamma[H(x_0)t + Gvt^2/2]$$

where $G = (dH/dx)_{x_0}$.

We see that the presence of a velocity v produces a phase shift in time t given by $\Delta\varphi = \gamma Gvt^2/2$ (assuming a constant field gradient). In practice, the transport effect can be measured by observing the constructive interference of the various spin volume elements distributed throughout x_0. Actually x_0 should pertain to a volume, but an analysis in one dimension is sufficient to give proper orders of magnitude.

Electronic apparatus is necessary here in order to measure spin echoes [*Hahn*, 1950]. A coil of sufficient volume (a few liters or more, as desired) is immersed and fixed in the sea. As in the Varian magnetometer [*Packard and Varian*, 1954], protons precess in the earth's field after an initial polarizing field is turned off at $t = 0$. In a time interval from $t = 0$ to $t = \tau$, let the protons precess in a total magnetic field made up of the earth's field H_e, assumed to be perfectly homogeneous, and the inhomogeneous field $H(x)$, supplied by an appropriate second coil carrying a current I. At time $t = \tau$, the current I is reversed to $-I$, and $H(x)$ changes to $-H(x)$.[1] Under these conditions, we solve for the spin echo. It is convenient to assume that the fraction of spins which precess in an inhomogeneous field $H(x_0)$ at x_0 is given by

$$P(\Delta\omega) = N \exp(-\Delta\omega^2/2\Delta\omega_{av}^2)$$

where N is a normalizing coefficient, $\gamma H(x_0) = \Delta\omega$, $\gamma H_e = \omega_e$, and $\Delta\omega_{av}^2$ is the mean square deviation in Larmor frequency due to $H(x_0)$. First we compute the phase behavior of $V(t)$ in the time intervals 0 to τ and from τ to t; we then average $V(t)$ over $P(\Delta\omega)$ and look for constructive interference from spin echoes at some time t_e for $t > \tau$.

At $t = \tau$,

$$V(\tau) = \exp[j(\omega_e + \Delta\omega)\tau + j\gamma Gv\tau^2/2]$$

For $t \geq \tau$, and if we note that $\Delta\omega$ and G change sign (not the G above) because current I is reversed,

* On leave of absence from the Department of Physics, University of California, Berkeley 4, Calif.

[1] Instead of reversing I, a radiofrequency pulse applied for a time t_w at the average Larmor frequency ω_e would serve the same purpose. If the field amplitude of the pulse is H_1, then $\gamma H_1 t_w = \pi$ is the necessary condition.

$$V(t) = \exp(+j\omega_e t)\cdot\exp\Big[-j\Delta\omega(t-2\tau) + \frac{j\gamma G v\tau^2}{2} - j\gamma G v\int_\tau^t t\,dt\Big]$$
$$= \exp\Big[j\omega_e t - j\Delta\omega(t-2\tau) - j\gamma G v\Big(\frac{t^2}{2}-\tau^2\Big)\Big]$$

The measured or average value of $V(t)$ is

$$\overline{V(t)} = \int_{-\infty}^{\infty} P(\Delta\omega)\,V(t,\Delta\omega)\,d\Delta\omega$$
$$= N(2\pi\overline{\Delta\omega_{av}^2})^{1/2}\exp\Big[j\omega_e t - j\gamma G v\cdot\Big(\frac{t^2}{2}-\tau^2\Big)\Big]\exp\Big[-\frac{(t-2\tau)^2\Delta\omega_{av}^2}{2}\Big]$$

where $N(2\pi\,\Delta\omega_{av}^2)^{1/2} = 1$. A signal maximum occurs essentially at $t = 2\tau = t_e$, and the phase shift of this signal, due to the velocity v, is

$$\Delta\varphi = \gamma G v\tau^2$$

The problem now is to measure $\Delta\varphi$ for a given minimum v to be detected. Assume $G = 0.01$ gauss/cm.[3]

[3] To some extent a larger G value could be used but $V(t)$ then attenuates because of molecular self-diffusion [see also *Herzog and Hahn*, 1956 (appendix)]. From spin-echo experiments, this produces an attenuation of signal amplitude given by

$$\exp[-2/3(\gamma G)^2Dt^3]$$

where D is the self-diffusion coefficient. If we let $t = 1$ sec, and $D = 2\times10^{-5}$ cm/sec^2 for water, then attenuation, due to self-diffusion, to $1/e$ of the initial amplitude, is given when $G \cong 0.01$ gauss/cm. We therefore limit G to this value.

Let $\tau = 1$ sec, which is approximately the relaxation time for sea water, and let $\gamma = 2.7\times10^4$ for protons. If we wish to detect a velocity $v = 10^{-3}$ cm/sec, then $\Delta\varphi \cong 0.3$ radians. This phase shift could be measured simply by beating $V(t)$ against a dummy echo signal from an identical apparatus, where $v = 0$. In this way it is possible to cancel out fluctuations in $H(x)$ (or I) which would otherwise cause phase shifts exceeding $\Delta\varphi$. The echo signal will last for a time

$$\Delta t = 2\pi/(\Delta\omega_{av}^2)^{1/2} = 2\pi/\gamma Gl \approx 10^{-3}\text{ sec}$$

where $l \cong 30$ cm is chosen as the sample breadth. This should be sufficient time for resolution, since it allows an observed free precession at the average frequency ω_e through at least 10 radians.

If two fixed stations are separated over long distances from one another and each station carries out simultaneous measurements, it would be possible to correlate velocities v at the two positions by noting correlations in phase shifts $\Delta\varphi$. Also the sign of $\Delta\varphi$ would reflect the sign of v. It does not appear feasible to carry out the proposed measurement on a moving platform unless means can be found for correcting or canceling out fluctuations of platform motion to a high degree.

References

Hahn, E. L., Spin echoes, *Phys. Rev.*, *80*, 580–594, 1950.

Herzog, B., and E. L. Hahn, Transient nuclear induction and double nuclear resonance in solids, *Phys. Rev.*, *103*, 148–166, 1956.

Packard, M., and R. Varian, Free nuclear induction in the earth's magnetic field, *Phys. Rev.*, *93*, 941, 1954.

(Received September 12, 1959.)